Introduction to Risk, Return and Opportunity Cost of Capital

Thomas Emilsson, 2010-10-15

Risk, Return and Portfolio Variance

Risk and Return

When making investments, there needs to be an estimation of the risks and returns that can be realized.
Risk is basically described as a measure of the uncertainty of returns in an investment, while returns are the cash flows from the investment.

Investors are always concerned about the link between risk and return in a potential investment. The return on your money will always be relative to the risk involved.
In other words, lower risk provides more certainty with lower returns, whereas higher risk has more volatility with higher returns.

Portfolio and Variability

An investment portfolio is defined as the sum total of a firm's or investor's assets. This could include stocks, bonds, futures, options, real estate, etc. Every portfolio carries a certain amount of risk depending what they are invested in. This is known as portfolio risk.

Statistics are employed to measure portfolio risk. The most important aspect when dealing with risk is to know how to accurately measure variability. Variability is simply put a measure of uncertainty.

When calculation variability, probabilities are assigned to each deviation which are calculated to obtain one single value called variance.

The other statistical tool we use is derived from the variance. The standard deviation is easily calculated by taking the square root of the variance. The standard deviation is by far the most widely used method of measuring variability and is defined as a measure of dispersion of returns from the expected return.

You can estimate the variability of any portfolio of stocks or bonds using the procedure described in the following example.

Rate of Return, Variance and Standard Deviation

Example 1:

You are offered to play in a game of chance that offers the following odds and payoffs. Each play of the game costs $100, so the net profit per play is the payoff less $100.

Probabillity	Payoff	Net Profit
.10	500	400
.50	100	0
.40	0	-100

What is the expected rate of return?

Expected return = (.10 x 400) + (.5 x 0) + (.4 x -100) = 0%

What is the variance of this rate of return?

$\sigma^2 = (400 - 0)^2 \times .10 + (0 - 0)^2 \times .5 + (-100 - 0)^2 \times .4 = 20,000$

What is the standard deviation of this rate of return?

$\sigma = \sqrt{20000} = 141$

The standard deviation is in the same units as the rate of return, meaning that the game's standard deviation is 141%. This is a very high standard deviation and therefore not a game you should take part in.

Example 2:

This time you are offered to play the following game.
You start by investing $100 Two coins are flipped. If both coins turn up heads, you get $140. If both coins turn up tails, you get $80 back. If the coins each show something different, you get $110 back.

(1) Percent Rate of Return	(2) Deviation from Mean	(3) Squared Deviation
+ 40	+ 30	900
+ 10	0	0
+ 10	0	0
- 20	- 30	900
Variance = average of squared deviations = 1800/4 = 450		
Standard deviation = square of root varia nce = $\sqrt{450}$ = 21.2%		

What is the expected rate of return?

Expected return = (.25 x 40) + (.25 x -20) + (.5 x 10) = 10%

What is the variance of this rate of return?

$\sigma^2 = (40 - 10)^2 \times .25 + (10 - 10)^2 \times .5 + (-20 - 10)^2 \times .25 = 450$

What is the standard deviation of this rate of return?

$\sigma = \sqrt{450} = 21$

This game's expected return is 10%, the variance of the percentage return is 450 and the standard deviation is the square root of 450, or 21. This is the same unit as the rate of return, so the game's variability is 21%.

Summary

These two examples is a simple way of showing how to calculate variance and standard deviation. In principle, you could estimate the variability of any portfolio of stocks or bonds by the procedure above.

The risk of an asset can be completely expressed by all possible outcomes and the probability of each. In practice, this is a little harder to do, therefore variance and standard deviation is utilized for this purpose. We must for that reason observe the past when predicting the future. It is reasonable to assume that portfolios with a history of high variability have the least predictable future performance.

Group work, Diversification and Portfolio Risk

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Case: Calculations

Diversification and Portfolio Risk

Diversification reduces risk in a portfolio of stock. To fully understand the effect of diversification, we need to know the risk of the portfolios individual shares.

We will here use an example of a portfolio consisting of stocks from the U.S stock market. In the portfolio we will have two stocks, 60% is invested in Wal-Mart and the remainder is invested in Boeing. Over the coming year we expect that Wal-Mart will give a return of 10% and Boeing, 14%.

The expected return of this portfolio is simply a weighted average of the expected returns on the individual stocks:

Question 1: What is the expected return of this portfolio?

When you have calculated the expected return of your portfolio it is then time to calculate the risk of this portfolio.

In the past the standard deviation of return was 19.8% for Wal-Mart and 29.8% for Boeing. In this case we assume a correlation coefficient of 1.0. We believe that these figures are a good representation of possible future outcomes.

Question 2: What is the variance of this portfolio?

Question 3: What is the standard deviation of this portfolio?

	Wal-Mart	Boeing
Wal-Mart	$x_1^2\sigma_1^2$	$x_1x_2\sigma_{12} = x_1x_2\rho_{12}\sigma_1\sigma_2$
Boeing	$x_1x_2\sigma_{12} = x_1x_2\rho_{12}\sigma_1\sigma_2$	$x_2^2\sigma_2^2$

In the next exercise we assume that Boeing and Wal-Mart do not move in perfect lockstep. In this case we will use the correlation coefficient of .35.

Question 4: What is the variance of this portfolio with a correlation coefficient of .35?

Question 5: What is the standard deviation of this portfolio?

	Wal-Mart	Boeing
Wal-Mart	$x_1^2\sigma_1^2$	$x_1x_2\sigma_{12} = x_1x_2\rho_{12}\sigma_1\sigma_2$
Boeing	$x_1x_2\sigma_{12} = x_1x_2\rho_{12}\sigma_1\sigma_2$	$x_2^2\sigma_2^2$

Question 6: In which of the following situations would you get the largest reduction in risk by spreading the above investment across these two stocks?

a. The two shares are perfectly correlated.
b. There is no correlation.
c. There is modest negative correlation.
d. There is perfect negative correlation.

Case: Calculations, Diversification and Portfolio Risk

Answers

1. What is the expected return of this portfolio?

Expected portfolio return = (.60 x 10) + (.40 x 14) = **11.6%**

2. What is the variance of the portfolio with a correlation coefficient of 1.0?

	Wal-Mart	Boeing
Wal-Mart	$x_1^2\sigma_1^2 = (.60)^2 \times (19.8)^2$	$x_1x_2\rho_{12}\sigma_1\sigma_2 = .40 \times .60 \times 1 \times 19.8 \times 29.8$
Boeing	$x_1x_2\rho_{12}\sigma_1\sigma_2 = .40 \times .60 \times 1 \times 19.8 \times 29.8$	$x_2^2\sigma_2^2 = (.40)^2 \times (29.8)^2$

= [(.6)² x (19.8)²] + [(.4)² x (29.7)²]
+ 2(.6 x .4 x 1 x 19,8 x 29.8) = **566.4**

3. What is the standard deviation of this portfolio?

   The standard deviation is the square root of the portfolio variance.

   $\sigma = \sqrt{566.4} = \mathbf{23.8\%}$

4. What is the variance of the portfolio with a correlation coefficient of .35?

	Wal - Mart	IBM
Wal - Mart	$x_1^2\sigma_1^2 = (.60)^2 \times (19.8)^2$	$x_1 x_2 \rho_{12}\sigma_1\sigma_2 = .40 \times .60 \times .35 \times 19.8 \times 29.8$
IBM	$x_1 x_2 \rho_{12}\sigma_1\sigma_2 = .40 \times .60 \times 1 \times 19.8 \times 29.8$	$x_2^2\sigma_2^2 = (.40)^2 \times (29.8)^2$

$= [(.6)^2 \text{ x } (19.8)^2] + [(.4)^2 \text{ x } (29.8)^2]$
$+ 2(.6 \text{ x } .4 \text{ x } .35 \text{ x } 19.8 \text{ x } 29.8) =$ **382.3**

5. What is the standard deviation of this portfolio?

   The standard deviation is the square root of the portfolio variance.

   $\sigma = \sqrt{382.3} = \mathbf{19.6\%}$

6. In which of the following situations would you get the largest reduction in risk by spreading the above investment across these two stocks?

   Answer: **d**

   When two stocks are perfectly negatively correlated, there is always a portfolio strategy that will completely eliminate risk. This is something that in common stocks really never occur.