Introduction to Risk, Return and the Opportunity Cost of Capital

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Definitions and Formulas

Investment risk

There are three basic questions arising when we start thinking about investment risk. The first question arising is how the risk is defined; the second, how it should be calculated for an investment; and the third examines how to prevent or lower the risk for an investment. Intuitionally, we would answer, first, risk is the possible difference between our expected outcome and the real outcome in the future. Second, we should compare our investment with other investments done in the past to calculate how risky it is. And third, we should diversify our investment instead of putting all our eggs in one basket. Surprisingly, that is exactly how it is done. Before we start measuring risk, we will introduce the mathematical basics needed to do so.

Arithmetic Averages and Compound Annual Returns

Arithmetical average of n annual rates of return r_1 to r_n :

$$\frac{r_1 + r_2 + ... + r_n}{n}$$

Compound (geometric) average of n annual rates of return:

$$\sqrt[n]{(1+r_1) \cdot (1+r_2) \cdot ... \cdot (1+r_n)}$$

When we speak of averages, we mean, if not otherwise stated, arithmetical averages. Without proofing, we assume, that if the cost of capital is estimated from historical returns or risk premiums, arithmetical averages correctly measure the cost of capital as opposed to compound averages.

Variance, Standard Deviation, Covariance

Our first basic assumption for the stock market is that the rates of return are normally distributed.
The *variance* of the market return is defined as:

$$\sigma_{\tilde{r}_m}^2 = \text{the expected value of} (\tilde{r}_m - r_m)^2,$$

$\tilde{r}_m$ denotes the actual return and r_m the expected return.

Variance estimated from a sample of n returns:

$$\sigma^2_{\tilde{r}_m} = \frac{1}{n-1}\sum_{t=1}^{n}(\tilde{r}_{mt} - r_m)^2$$

where $\tilde{r}_{mt}$ is the market return in period t and r_m is the mean of the values of $\tilde{r}_{mt}$.

The *Standard Deviation* is the square of the variance:

$$\sigma_{\tilde{r}_m} = \sqrt[2]{\sigma^2_{\tilde{r}_m}}$$

The *covariance* between stocks 1 and 2 is defined as:

$$\sigma_{12} = \rho_{12}\sigma_1\sigma_2,$$

where ρ_{12} denotes the correlation factor between stocks 1 and 2.

Portfolio Variance and Beta

Holding a portfolio of stocks, the *portfolio variance* is:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}x_i x_j \sigma_{ij}$$

where x_n denotes the proportion of stock n compared to the portfolio.

The *beta* relative to market portfolio (or, more simply, *beta*) is defined as:

$$\beta_i = \frac{\sigma_{im}}{\sigma^2_m},$$

where σ_{im} is the covariance between stock and the market returns and σ^2_m is the variance of the returns on the market.

Risk premium and Historical Review of Capital Markets

A Century of Capital Market History

We have taken a look at the performance of two portfolios of U.S. securities since 1900, information based on a study by Dimson, Marsh, and Staunton.

1. A portfolio of Treasury bills, ie, U.S. government debt securities maturing in less than one year (before 1919 the commercial paper rate is used in this study).
2. A portfolio of U.S. common stocks.

Treasury bills offer an investment as safe as you can make, without a risk of default, and relatively stable prices caused by the short maturity. The only uncertainty is the inflation and the resulting *real* rate of return. We only focus on the *nominal* rate of return, so we need not bother about inflation; we will simply keep it in mind. The consequence of investing in common stocks

is that we share all of the up- and downswings inherent to them. The average annual rate of return from 1900 to 2006 was about 4% for U.S. treasury bills and 11.7% for U.S. common stocks. We define the extra return of common stocks versus treasury bills (7.6% in this case due to rounding) as the average (historical) *risk premium*. You may ask why we look back such a long period and why we only focus on the U.S. market to define our historical risk premium.

The only reasonable approach to correctly measuring annual rates of return on common stocks is to look at a very extensive period – due to the high fluctuation. Furthermore, the afore-mentioned study shows that our risk premium is about average compared with evaluated risk premiums in other industrial nations for the same period of time.

Using Historical Evidence to Evaluate Today's Cost of Capital

How to estimate the cost of capital today? Let's suppose there is an investment project as risky as the Standard and Poor's Composite Index. We will suppose, to take things a bit loosely, it has the same degree of risk as the market portfolio. What rate should you use to discount this project's forecasted cash flows? Clearly you should use the currently expected rate of return on the market portfolio. This will be termed market return r_m.

Assuming that the future will be comparable to the past and that today's investors expect the same "normal" rates of return as in the past, you would set r_m at 11.7%, the average of past market returns. Do you believe that investors would be content to hold common stocks offering an expected return of 11.7% when Treasury bills were offering 15% – as they did in 1981, for example? That's why r_m is defined as the sum of the risk-free interest rate and the risk premium.

An example: In mid 2006, Treasury bills were about 5%. Adding on the risk premium 7.6% yields
$$r_m(2006) = 5\% + 7.6\% = 12.6\%$$

The crucial assumption here is that there is a normal, stable risk premium on the market portfolio, so that the expected *future* risk can be measured by the average past risk premium.

Even with over 100 years of data, we can't estimate the market risk premium; nor can we be sure that investors today are demanding the same reward for risk that they were 50 or 100 years ago. All this leaves plenty of room for arguments about what the risk premium *really* is. Some reasons why today's expected risk premium might be less than the historical average are the fact that today the economies and markets seem to be more stable than they were in the past and today's investors can more easily diversify their investments, thus diminishing their

risk. There are few who would subscribe to the notion that today's risk premium is higher than the historical one.

Dividend Yields and the Risk Premium

Another way to determine the risk premium is to have a closer look at past dividend yields. If stock prices are expected to keep pace with the growth in dividends, then the expected market return is equal to the dividend yield plus the expected dividend growth. Dividend yields in the United States have averaged 4.4% since 1900, and the annual growth in dividends has averaged about 5.6%. The expected market value return over this period was

$$r = \frac{DIV_1}{P_0} + g = 4.4 + 5.6 = 10\%$$

This is 6% above the risk-free interest rate and 1.6% lower than our *realized* risk premium before. Furthermore, a reduction in the dividend yields would appear to herald a reduction in the risk premium that investors can expect over the subsequent few years. Therefore when yields are relatively low, companies may be justified in shaving their estimate of required returns over the next few years. However, changes in the dividend yield tell companies next to nothing about the expected risk premium over the next 10 or 20 years.

There is only one firm conclusion emerging from this debate: Do not trust anyone who claims to *know* what returns investors expect.

Many financial economists rely on the evidence of history and work with a risk premium of 7.5%. Others use a somewhat lower figure.

Portfolio Risk

You have already learned the discount rate for safe projects and for average-risk projects. But you still *don't* know how to estimate discount rates for assets that do not fit into these straightforward categories.

Measuring Variability

In principle, you could estimate the variability of any portfolio of stocks or bonds using the procedure described in the following example. You would identify the possible outcome, and do the number-crunching calculations.

> For example: Suppose you are offered to play the following game. You start by investing $100. Then two coins are flipped. If both coins turn up heads, you get $140 back. If both coins turn up tails, you get $80 back. Or if they each show something different, you get $110 back.
> Exp. return $= (.25 \cdot 40) + (.25 \cdot (-20)) + (0.5 \cdot 10) = 10\%$

$$\sigma^2 = (40-10)^2 \cdot .25 + (10-10)^2 \cdot .5 + (-20-10)^2 \cdot .25 = 450$$
$$\sigma = \sqrt{450} \approx 21$$

This game's expected return is 10%, the variance of the percentage return is 450 and the standard deviation is 21. The standard deviation is in the same units as the rate of return, meaning that the game's variability is 21%.

The risk of an asset can be completely expressed by all possible outcomes and the probability of each. In practice, this is often impossible, therefore variance and standard deviation are used for summarizing the spread of possible outcomes. Unfortunately, the probability of each outcome is not fixed and cannot be found in a newspaper or elsewhere. So we must resort to observing the past once again. Of course, hindsight entails no risk, but it is reasonable to assume that portfolios with histories of high variability also have the least predictable future performance. The annual standard deviation for Treasury bills from 1900-2006 was 2.8 and 19.8 for common stocks. Just as today's premium risk may be different from historical risk, today's market variability may be different as well.

Diversification

The standard deviation of the U.S. market portfolio during the five-year period from July, 2001 – June, 2006 was about 16%. Wouldn't you expect that the standard deviation of one single stock should be close to 16% on average for the same period? – ie, the standard deviation of one single stock is higher than the standard deviation of the market portfolio because *diversification reduces variability*. Diversification works because stocks are not perfectly correlated, meaning prices of different stocks do not move in exactly the same ways. Even a little diversification can provide a substantial reduction in variability.

> For example: The standard deviation for IBM during the five-year period of July, 2001 – June, 2006 was about 29.7% and for Boeing about 29.8%. The standard deviation of a portfolio evenly distributed between IBM and Boeing was about 21.5%. This is 8% less than holding just one of those stocks.

The improvement is much smaller when the number of securities is increased beyond, say 20 or 30. The risk that can be potentially eliminated by diversification is called *unique risk* (or unsystematic, residual, specific, diversifiable risk). Unique risk stems from the fact that many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors. But there is also some risk that you cannot avoid regardless of how much you diversify.

This risk is called *market risk* (or systematic, undiversifiable risk). Market risk stems from the fact that there are perils that threaten all businesses. That is why stocks have a tendency to move together.

Calculating Portfolio Risk

We now have an intuitive idea of how diversification reduces risk. The next step is to make a formulary definition of portfolio variance and to practice some calculations. The formula for portfolio variance is:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

Inserting the covariance for stock i and stock j ($\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$):

$$\text{Portfolio variance} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \rho_{ij}\sigma_i\sigma_j$$

To calculate portfolio risk we need to know the correlation coefficients of the stocks and their single standard deviation.

> For example: Suppose that 60% of your portfolio is invested in Wal-Mart (stock 1) and the remainder is invested in IBM (stock 2). You expect that over the coming year Wal-Mart will give a return of 10% and IBM, 15%. The expected return, r_p, on your portfolio is:
>
> $$r_p = (.6 \cdot 10) + (.4 \cdot 15) = 12\%$$
>
> In the past, the standard deviation of IBM was about 29.7% and for Wal-Mart about 19.8%. The past correlation was about .35. You believe that these figures are a good representation of the spread of possible future outcomes.
>
> $$\sigma_p = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \rho_{ij}\sigma_i\sigma_j = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12}\sigma_1\sigma_2)^2$$
>
> $$= (.6)^2 \cdot (19.8)^2 + (.4)^2 \cdot (29.7)^2 + 2(.6 \cdot .4 \cdot .35 \cdot 19.8 \cdot 29.7)^2$$
> $$= 381.1$$
>
> $$\sigma = \sqrt{381.1} \approx 19.5\%$$
>
> The expected standard variation is 19.5%. Compared to investing in Wal-Mart only, we reduced risk and improved our expected return by adding a stock with a higher risk and a higher expected return due to diversification.

How Individual Securities Affect Portfolio Risk

The risk of a well-diversified portfolio depends on the market risk of the securities included in the portfolio. As we mentioned before, the risk of a security is the sum of unique risk and market risk. While unique risk can be eliminated by diversification, market risk cannot. Market risk of a security is the sensitivity to market movements, called *beta* (β). The beta

of the market portfolio is 1.0. A stock with a beta higher than 1.0 tends to amplify the overall movements of the market. A stock with a beta between 0 and 1.0 tends to move in the same direction as the market but not as far. Often stocks with high standard deviation have high betas. The risk of a well-diversified portfolio is proportional to the portfolio beta, which equals the average beta of the securities included in the portfolio. Hence, the risk of a well-diversified portfolio with a beta of 0.5 is half of the risk of the market portfolio and a third of the risk of a well-diversified portfolio with a beta of 1.5.

> For example: The following table shows the returns over a particular sixth-month period on the market and the stock of Anchovy Queen restaurant chain. Although both investments provided an average return of 2%, Anchovy Queen's stock was particularly sensitive to market movements, $\beta = 1.5$. A diversified portfolio of stocks with the same beta as Anchovy Queen would be one-and-a-half times as volatile as the market.

Month	Market return	Anchovy Q return	Deviation from average market return	Deviation from average Anchovy Q return	Squared deviation from average market return	Product of deviations from average returns
1	-8%	-11%	-10	-13	100	130
2	4%	8%	2	6	4	12
3	12%	19%	10	17	100	170
4	-6%	-13%	-8	-15	64	120
5	2%	3%	0	1	0	0
6	8%	6%	6	4	36	24
Average	2%	2%		Total	304	456
Variance = σ_m^2 = 304/6 = 50.67						
Covariance = σ_{im} = 456/6 = 76						
Beta (β) = σ_{im}/σ_m^2 = 76/50.67 = 1.5						

Calculating the variance of the market returns and the covariance between the returns on the market and those of Anchovy Queen. Beta is the ratio of the variance to the covariance.

Diversification and Value Additivity

Diversification reduces risk and, therefore, makes sense for investors. Undoubtedly, diversification is a good thing but that does not mean that firms should practice it. As long as investors have the easy option, as they do today, to diversify on their own account, they do not pay any more for a firm that offers them diversification. An individual can invest in the steel industry this week and pull out next week. A firm cannot do that. To be sure, the individual would have to pay brokerage fees on the purchase and sale of steel company shares, but think of the time and expense for a firm to acquire a steel company or to start up a new steel-making operation. Another aspect is that if investors have a sufficiently broad choice of securities they will not pay any less because they are unable to invest separately in each factory.

In countries which have large and competitive capital markets diversification does not add or subtract from the value of a firm. The total value is the sum of its parts. This concept is called *value additivity*. If the capital market establishes a value PV(A) for asset A and PV(B) for asset B, the market value of a firm holding only these two assets is

$$PV(AB) = PV(A)+PV(B)$$