Other Ways to Adjust for Risk

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Certainty Equivalents

When using the certainty equivalent, your payoff is less than the expected value of the investment because you prefer to reduce the uncertainty. You would have to weigh between the expected value of the investment and a given payoff.

Valuation

There are two ways of taking account of risk when calculating the present value:
You can discount the expected cash flow C_1 by the risk-adjusted discount rate r:

$$PV = \frac{C_1}{1+r}$$

Or you can discount the certainty equivalent of a safe cash flow CEQ_1 by the risk-free rate of interest r_f.

$$PV = \frac{CEQ_1}{1+r_f}$$

For cash flows over two, three or t years away:

$$PV = \frac{C_1}{(1+r)^t} = \frac{CEQ_1}{(1+r_f)^t}$$

Without proof, the equivalent certainty-equivalent form of the CAPM states:

$$CEQ_1 = C_1 - \lambda \operatorname{cov}(\tilde{C}_1, \tilde{r}_m)$$

where $\operatorname{cov}(\tilde{C}_1, \tilde{r}_m)$ is the covariance between the uncertain cash flow, $\tilde{C}_1$, and the return on the market, $\tilde{r}_m$. Lambda, $\lambda = (r_m - r_f)/\sigma_m^2$, is a measure of the market price of risk.

Proper Use of a Single Risk-Adjusted Discount Rate for Long-lived Assets

In this paragraph we will show when a constant risk-adjusted discount rate can be used to calculate a present value. Furthermore, we explain why distant cash flows should *not* be discounted at a higher rate than earlier cash flows.

For example: Let us consider two simple projects. Project A is expected to produce a cash flow of $100 million for each of three years. The risk-free interest rate is 6%, the market risk premium is 8%, and project A's beta is 0.75. Therefore, you calculate A's opportunity cost of capital as follows: $r = r_f + \beta(r_m - r_f) = 6 + .75 \cdot 8 = 12\%$

Project B produces three cash flows: 94.6 million in the first, 89.6 million in the second and 84.8 million in the third year. All of them are supposed to be safe, meaning they should be discounted by the risk-free interest rate of 6%. Now compare the present value of these projects' cash flows:

$$PV_A = \frac{100}{1.12} + \frac{100}{(1.12)^2} + \frac{100}{(1.12)^3} = 89.3 + 79.7 + 71.2 = 240.2$$

$$PV_B = \frac{94.6}{1.06} + \frac{89.6}{(1.06)^2} + \frac{84.8}{(1.06)^3} = 89.3 + 79.7 + 71.2 = 240.2$$

Since the present value of the future cash flows are the same, we could say that project B has the same certainty equivalent as project A. To get rid of the uncertainty in project A, we need to give up parts, increasing with time, of our future cash flows.

When using a constant rate, the deduction for risk from later cash flows increases. Therefore, a constant risk-adjusted discount rate should be used for a stream of cash flows, if we assume that risk accumulates at a constant rate when we calculate for a longer period. Due to this effect, it would be wrong to discount future cash flows to a greater extent than earlier cash flows only because of their timing – as we already implied a greater deduction for later cash flows.

Limits of Single Risk-Adjusted Discount Rates for Long-lived Assets

Unfortunately, a lot of investments, especially options, include continually changing risk. Ignoring this fact and using a single risk-adjusted discount rate, instead of the certainty-equivalent method, will very often result in our making the wrong decision. This example shows how wrong our decision could be:

For example: Vegetron has to make a decision on the following project. Scientists developed an electric mop. The preliminary phase (testing and pilot production) would last one year and costs $125,000. If the preliminary phase is a success, Vegetron will build a plant that would generate $250,000 per year after taxes. If it is not a success the project will have to be dropped. Management has little experience in consumer products and feels that there is only a 50% change for a successful preliminary phase. They consider this a project of extremely high risk. (We will assume that they mean high market risk) Therefore management discounts the cash flows at 25%, rather than at Vegetron's usual 10% standard:

$$NPV = -125 - \frac{0.5 \cdot 1000}{1.25} + 0.5 \cdot \sum_{t=2}^{\infty} \frac{250}{(1.25)^t} = -125$$

It would thus seem that the project is not worthwhile.
If the considered high risk of this project depends (strongly or not) on the variability of the preliminary phase, the management analysis will be wrong. If the test phase is a failure, then there is no risk at all. And if the test phase is a success, there could well be just normal risk, discounted by the normal 10%, from then on. Or maybe another firm wants to purchase this project, discounting it at 10%. We are just splitting the first project into two different projects:

Project A, the possible plant in one year:

$$NPV_A = -1{,}000 + \frac{250}{.10} = +1{,}500$$

Project B, the preliminary phase, offering an expected payoff of $.5 \cdot (1{,}500) + .5 \cdot 0 = 750$, or $750,000, at year one on an investment of $125,000. Of course, the certainty equivalent of the payoff is less than $750,000, for example, half the forecasted cash flow and the risk-free rate is 7%:

$$NPV_B = C_0 + \frac{CEQ_1}{1+r} = -125 + \frac{.5 \cdot 750}{1.07} = 225.5$$

Project B, and also our first project is calculated to be worth $225,500. Thus, by taking a wrong approach towards calculating the project, the management would have rejected a promising project.

International Projects

Risk of Foreign Investment

CAPM can help to estimate the cost of capital for domestic investments by U.S. companies. But can the procedure be extended to investments in other countries? The answer is yes although there are some fixes needed to take the other countries' economy and market situations into consideration. At first sight, an investment seems to be *riskier* when it is a common stock of a foreign country as opposed to one on the U.S. stock market. Many countries have a higher ratio of standard deviation on their domestic index compared to Standard & Poor's Composite Index.

Betas of nine country indexes, calculated from monthly returns, February 2002-Januar 2007			
	Ratio of Standard Deviations[1]	**Correlation Coefficient**	**Beta**[2]
Argentina	2.36	.32	.75
Brazil	2.10	.64	1.34
China	1.96	.02	.04
Egypt	1.49	.09	.14
India	1.80	.39	.70
Indonesia	1.71	.34	.58
Mexico	1.36	.68	.93
Sri Lanka	2.07	-.06	-.13
Turkey	2.96	.53	1.57

[1]Ratio of standard deviations of country index to S&P Index.
[2]Beta is defined as β=1 for the U.S. market

But if we take a closer look on the Betas of these markets, we realize that, excepting Brazil and Turkey, all other countries have a beta lower than 1, due to low correlation.
We can conclude that the unique risk of a single stock in a foreign country is supposed to be higher than a stock on the U.S. market. However, the market risk, expressed by beta, is lower. Opportunity Cost of Capital should be measured by market risk and therefore it might be *safer* to invest in a foreign country than in the U.S. market.

Possibilities of Foreign Investment

Although we cannot make a general decision on where to invest, foreign country markets offer another possibility to diversify and therefore decrease risk. How this might work out is illustrated in the following example.

> For example: Suppose that Lafarge, the giant French producer of building materials, is considering a new plant in Bordeaux. The financial manager uses CAPM but this time she uses betas relative to the *French* market index and *euros* instead of dollars as the currency. Suppose her measures points out a beta of 0.94 (compared to the French market) and that the expected return on the French market is 7% above the euro interest rate. Then Lafarge needs to discount the euro cash flows from a new plant at $0.94 \times 7 = 6.6\%$ above the euro interest rate. But now suppose that Lafarge considers expanding into the United States. Once again, the financial manager measures beta relative to the French market index. But the fortunes of a U.S. plant will be less closely tied to the French index. One useful guide is the beta of U.S. building market stocks *relative to the French market*. This beta was about 0.54 between 2002 and 2006. If this estimate is right, then Lafarge should discount the euro cash flows from its U.S. project at $0.54 \times 7 = 3.8\%$ above the euro interest rate.

We have to be careful when measuring risk relative to a domestic market because we are indirectly assuming that a shareholder simply holds domestic stocks. If they do, a firm could, as in the example, accept a lower interest rate for an investment in a country whose market index is not perfectly correlated to the domestic market index – as would be the case with a comparable investment in the domestic country. Surprisingly, the above assumption seems to be true for shareholders in the United States. Although – as we mentioned before – internationally diversified portfolios offer a possibility to reduce risk, U.S. shareholders generally only invest a small proportion of their money overseas. No question about it – on the one hand there might be reasons for concern about investing in some other countries. On the other hand, however, there are a lot of reasons to do it. And investors everywhere are increasing their holdings of foreign securities. Perhaps in the near future, investors will hold internationally diversified portfolios and it will be advisable for firms to calculate betas relative to the world market. Then a firm should demand the same return from an investment in the United States as in France or in Egypt.

Lower Cost of Capital

Do lower rates of interest in one country include a lower cost of capital in the same country? Before we answer this question, we have to confront some problems. First of all, we should eliminate the effect of different currencies. The problem is not only the current exchange rate; it includes a hidden risk in the variability of the currencies, too. Furthermore, the countries' law and tax system may scatter our result. After taking nearly all possible problems into account, assuming we still have rates of interest that are *genuinely* lower, we are right to assume that the real cost of capital is lower in this country.

> For example: In 2007 the long-term interest rate in Japan was about 1.7%. In the United States it was about 4.8%. Someone might suppose that the Japanese companies enjoyed a lower cost of capital. Without determining the *real* long-term interest rate compared to the U.S. interest rate, the fact that financial institutions borrowed an estimated $200 billion in Japan and reinvested it at higher rates elsewhere, seems to underline the truth of this supposition.