Risk and Return

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Introduction

Every decision regarding investments is based on the relationship between risk and return. Generally the return on an investment should be as high as possible depending on the risk tolerance of the investor.
The aim of this essay is to give an overview of the relationship between risk and return in modern portfolio theory.

At first some typical terms are defined like different types of risk and the composition of the expected return of an asset and its relation to standard deviation. In the second part you will receive an overview on modern portfolio theory. A more detailed description is given for the *Capital Asset Pricing Model (CAPM)*. For further reading, a reference is listed in the last section.

Definitions

Types of risk and beta

The risk of any stock can be divided into:

- Unique risk
- Market risk

Unique Risk (Unsystematic risk) affects only a single stock or a small number of stocks. Unique risk describes a firm's specific risk related to the market. Uncertainties about a firm's labour contracts, suppliers or customers are part of unique risk. Investors can eliminate the unique risk by holding a well-diversified portfolio.
Market (Systematic) risk affects a large number of assets in the economy and is generally market wide. Uncertainties about the general economy, such as GDP, interest rates, inflation, etc. all affect systematic risk. Firms that produce for example goods with a long life expectancy, such as aircraft, are highly sensitive and thus have a high market risk. Conversely, firms that produce goods for daily needs have a lower market risk.
Market risk cannot be eliminated by holding a well-diversified portfolio; it is non-diversifiable.

Hence all risk of a fully-diversified portfolio is market risk which depends on its sensitivity to market changes.
Beta describes that sensitivity and is thus a measure of market risk. Beta measures the relationship between price movements of an individual stock to the market portfolio. A stock with a beta of 1.0 has an average market risk. A well-diversified portfolio of such stocks has the same standard deviation as the market index. A portfolio with stocks having a beta of 0.5 , has half of the standard deviation which means that the portfolio moves half as much as the market; it has a lower risk. If the beta of a stock is higher than one, it is riskier than the market. The beta of risk-free assets is 0.

Risk-Free Interest Rate and Risk Premium

Short-term Treasury bills (e.g. 3-month U.S. Treasury bills) can be seen as risk-free investments. They bring out benchmarks of ***risk-free interest rates***. As a result, the beta for risk-free assets is 0.
If an investor prefers to invest in an asset which is riskier than treasury bills, they are likely to demand a higher return.
In other words, an investment in risky assets like stocks should give a higher return than that of a risk-free asset. The difference between the market risk and the risk-free rate is called ***risk premium***. Risk premium represents the extra return (beyond the risk-free interest rate) investors demand for moving their funds away from a risk-free asset to a risky asset.
The risk premium should increase with the risk aversion of an investor and the average level of risk of the investment.

Expected Return, Standard Deviation and Correlation

Expected return is the weighted average of possible returns where the weights represent the probabilities of the returns.
Standard deviation describes the risk that expected return will or will not happen. This means that risk-free assets have a standard deviation of 0.
Correlation describes how strong different stocks' returns move together. Perfect positive correlation (where the correlation coefficient Corr =1) of two investments means that the returns on the investments always change at the same time in the same direction. Perfect negative correlation (Corr = -1) of two investments means that the returns on the investments always change at same time in the opposite direction. A correlation of zero (Corr = 0) means, that the returns are completely unrelated to each other.
Co-variance is a measure of co-movement between two random variables (how much two variables vary together). To calculate co-variance, you need to have the correlation coefficient.

Portfolio Theory

Modern portfolio theory is based on an article ***Harry Markowitz*** released in 1952. His ***Portfolio Selection Model*** describes how an investor can reduce the standard deviation of portfolio returns by choosing stocks that are correlated differently (Corr <1).
The principle of that portfolio selection model shall be explained in two easy examples, because they are the basis for the following theories.

Example 1:
If returns of stock are normally distributed, the expected return and the standard deviation are the only two measures that an investor needs to consider.
There are 3 investments.

- A: expected return 10%, standard deviation 15%
- B: expected return 10%, standard deviation 7.5%
- C: expected return 20%, standard deviation 7.5%

If investors should choose between A and B, they would prefer B, because of the lower spread of the expected return – the lower risk.
If investors should choose between B and C, they would prefer C, because of the higher expected return by the same risk (= same standard deviation).

Example 2:
There are 2 investments:

- A: expected return 12%, standard deviation 8%
- B: expected return 8%, standard deviation 4%

The investor can choose to invest only in A (higher expected return) or only in B (lower risk) or invest in a combination of both. The investment decision is depending on the risk tolerance of the investor.
The expected return of a portfolio is the weighted average of the expected returns of the two stocks. The portfolio risk is only the weighted average of the risks of the two stocks when the correlation is totally positive (Corr = 1).
If the correlation coefficient is totally negative (Corr = - 1), the risk of the portfolio is ‘0’. Meaning that diversification can lower the risk in portfolios if the correlation coefficient is lower than +1. Depending on the risk aversion of the investor and the correlation coefficient, there are many possibilities of building a portfolio of A and B.

In the portfolio theory ***efficient portfolios*** do exist.
Efficient portfolios are those who are not dominated by other portfolios. Their characteristics are that, there is no portfolio that:

- by the same expected return has a lower risk
- by the same risk has a higher expected return
- by a lower risk has a higher expected return

All efficient portfolios are along one graph – this graph is called the efficient line.
The portfolio theory gives a very good view into the attitude of risk-avers investors. That theory is the basis for the Capital Asset Pricing Model (CAPM).
The portfolio theory itself is not practical, because it is hard to estimate the correlation coefficient for two stocks and it is even harder to estimate when the portfolio consists of more than two correlation coefficients. In real life there is an almost unlimited range of possibilities of investing money – and not just in 2 assets, as mentioned in the above examples. This leads us to the next section.

The Capital Asset Pricing Model (CAPM)

The CAPM by John Lintner, Jack Treynor, and William Sharpe is based on the Portfolio Theory and was developed in the mid-1960s. CAPM explains which return investors would like to have for the bearing of risk. The perception behind the CAPM is the insurance motive of risk-averse investors.

Assumptions

- There is a competitive market and the market portfolio represents all assets in the economy.
- Existence of a risk-free investment. E.g. an investment in US Treasury bills is risk-free, meaning that beta is zero.
- Investors can borrow and lend money at the same interest rate.
- All investors are risk-avers and invest their money only in efficient Portfolios.
- All investors have the same expectations and information about risk and return.
- The market risk is measured by beta and the risk premium moves proportionally to its beta.

The main statement of the CAPM is that one can reduce risk by investing in a well-diversified portfolio.

CAPM should be explained with the following figure:

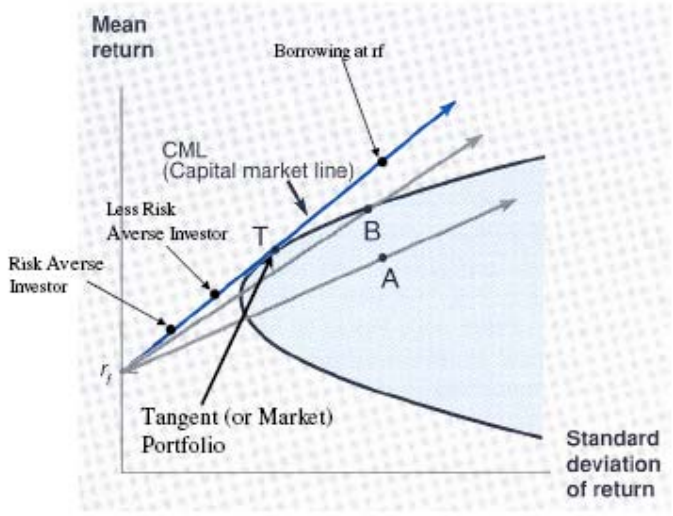

Figure: Capital Market line

In the CAPM we have more than two risky stocks. Therefore the number of possible portfolios is not along a graph, but within a field. The number of efficient portfolios is still on a graph, called the efficient frontier. Thus, the ***efficient frontier line*** *(part of the black graph)* is a graph representing a set of portfolios that maximizes expected return at each level of portfolio risk. Which portfolio on the efficient frontier the investor chooses depends on his risk tolerance.
In the CAPM it is assumed that an investor can choose portfolios which consist of a share of a risk-free asset like a treasury bill and a share of risky assets. The ***Capital market line*** (CML = blue graph) connects the risk-free asset with the risky market portfolio. This combination creates a greater set of possible efficient portfolios and a 'new' efficient frontier line. Because of the identical expectations and information, the capital market line is the same for all investors; therefore the CML describes all efficient portfolios of the market.
All investors independent of their risk aversion would be on the tangent line. Where they are on the tangent line will depend on the individual investor's risk tolerance.
If an investor is totally risk-averse they, would only invest in the risk-free assets (r_f) – beta is 0. On the CML between r_f and T, all investors would invest in a portfolio consisting of risk-free and risky assets. If an investor would like to take the risk belonging to point T, they would invest only in the risky market portfolio. In T beta is 1. If the investor has a higher risk tolerance than the risk belonging to T, they could invest borrowed capital additional to their equity in the market portfolio and would get a higher return on the CML (above T). However, the most efficient portfolio would always be T. There is no other portfolio that makes more sense in which to invest.

In the CAPM beta is used for the measurement of market risk to analyze the relation between risk and expected return. It is important to state that the risk premium of a stock only depends on the market risk, hence the higher the beta, the higher the market risk and the higher the expected return on the stock. Beta can be measured as the standardized covariance between the market return and return on a stock.

Conclusion of the CAPM

Diversification typically reduces risk. Only the non-diversifiable market risk - which is measured by beta - is important in pricing assets. CAPM combines linear risk-return trade-off with the beta to find the price of risk. The optimal risk-return trade-off is shown by the capital market line. Regardless of the risk tolerance an investor has, they have to choose a portfolio located along the capital market line, which shows them the best risk-return relation.

The Capital Asset Pricing Theory is the best-known model of risk and return. The model is logical und consistent.
But nevertheless the assumptions of the CAPM could be seen critical:

- ‘An investment in US Treasury bills is risk-free.’ Here the uncertainty about inflation is not considered.
- ‘Investors can borrow and lend money at the same interest rate.’ Generally borrowing rates are higher than lending rates.
- ‘All investors have identical expectations and investing in the market portfolio.’ This is far from reality.
- There are facts that are not considered in the CAPM like firm size, book-to market ratios, time of year.
- The market portfolio is unobservable so we need to use a proxy. Hence, CAPM is not testable. If you want to test it, in practice you can only test an index which represents the market.
- Past evidence has shown that in fact, expected returns increase with beta, but not as fast as the simple version of the CAPM predicts.

Regardless of the simple assumptions the CAPM is still the most important model for describing financial markets.

Alternative Models

Consumption CAPM

The CAPM has been modified for consumption. Instead of using a market beta for measuring risk, a *consumption beta* should be used. In that theory, the risk of the stock reflects the sensitivity of returns to changes in investor's consumption. The problem is that there are no sufficient methods of measuring consumption. That makes this model impracticable.

***Arbitrage pricing model* APT**

An alternative theory of risk and return is the *Arbitrage pricing theory (APT) by Stephen Ross.* The theory assumes that each stock's return depends partly on macroeconomic influences (factors) and partly on 'noise' (events that are unique to a company). The 'noise' as a unique risk of a company can be eliminated by diversification and has no influence on the expected return. Therefore the expected risk premium on a stock is only affected by macroeconomic risk (factor).

According to the APT the following steps should be made:

- Identification of macroeconomic factors that could affect the stock returns.
- Estimation of the expected risk premium on each of the factors $\mathbf{(r_{factor1}-r_f)}$
- Measurement of the sensitivity of each stock to the factor (b_1)

$$\textbf{Expected risk premium} = b_1(r_{factor1}-r_f) + b_2(r_{factor2} - r_f) + \ldots$$

The APT says that the expected return depends only on the risk arising from unexpected changes in the macroeconomic factors. In comparison to the CAPM the market portfolio plays no role in the APT. The expected return on a stock depends on more than just one market beta. For each stock there are several macroeconomic factors taken into account.
An advantage is that the market portfolio must not be measured. APT can be used when only data of a sample of risky assets is available. A disadvantage is that the different factors are not defined.

Fama-French Three-Factor Model

Fama and French have made research that identified factors that are related to company profitability – a risk factor that is left out in the CAPM. The new factors are the size of the company and the book-to-market factors. Both have an influence in earnings and returns. Fama and French showed that stocks of small firm's and those with high book-to-market-ratios have a high degree of

influence on returns – in the past these stocks had a higher return.

There are 3 steps using in the Three-Factor Model:

- Identification of the factor:
  - Market factor: $r-r_f$
  - Size factor: return on small less return on large firms stock
  - Book-to-Market-factor: return on high book-to-market ratio stocks less low book-to-market ratio stocks
- Estimation of the risk premium for each factor
  - r_{market}
  - r_{Size}
  - $r_{book\text{-}to\text{-}market}$
- Estimation of the factor sensitivities: stocks are differently sensitive to fluctuation in return on the three factors.

Expected risk premium = $b_{market}(r_{marketfactor}) + b_{size}(r_{sizefactor}) + b_{book\text{-}to\text{-}market}(r_{book\text{-}to\text{-}market})$

Conclusion of the Modern Portfolio Theories

Each of these different models has its pro's and con's and fans.

A conclusion is summarized as follows:

- Risk can be separated into market risk and unique risk.
- Unique risk of the company could be eliminated by diversification. Diversification reduces risk.
- Investors require an extra expected return for taking on risk. Therefore, expected return on an investment directly depends on its level of risk.
- Investors appear to be concerned predominantly with the risk that they cannot eliminate by diversification.
- The most frequently used model to describe financial markets and the relation between risk and return is still CAPM.

References

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