Risk and Return

Martin Glogger, 2008-09-30

Introduction

The relationship between risk and return is an essential factor in all human decision making. Each investment a firm undertakes, for example, must offer a return that is at least as high as the return on a similarly risky investment on financial markets. Otherwise shareholders would choose to invest in the financial markets rather than in the firm.
The aim of this essay is to give students an overview of the relationship between risk and return in modern portfolio theory.

After a conclusion in the first section, there are definitions given about the types of risk and the composition of the expected return of an asset and its relation to standard deviation. The second section will give an overview on modern portfolio theory and in particular on how the *Capital Asset Pricing Model* works. For further reading, some references are listed in the last section.

Conclusion

A pre-conclusion can be summarized as follows:

- Risk can be separated into market and unique risk.
- Expected return consists of a risk-free rate and a risk premium.
- Therefore expected return on an investment directly depends on its riskiness.
- Diversification reduces risk; by holding the market portfolio, one can eliminate unique risk.
- There are some alternatives but the most frequently used model to describe financial markets is still CAPM
- Regardless of the risk tolerance an investor has, he/she has to choose a portfolio located along the efficient frontier line, which shows the best risk-return relation.
- Combining the market portfolio with the possibility to borrow or lend money there is just one efficient portfolio investors should choose to hold.
- CAPM is just a model with simple premises, but it is still the most important model to describe financial markets.

Definitions

Types of risk

There are two different types of risk. We need to distinguish between:

- Market risk and
- Unique risk.

Market (Systematic) risk affects a large number of assets in the economy and is generally market wide. Uncertainties about the general economy, such as GDP, interest rates, inflation, etc. affect systematic risk. Market risk cannot be eliminated by holding a well-diversified portfolio; it's nondiversifiable. Firms that produce for example long-living goods such as aircraft are highly sensitive and thus have high market risk. Conversely, firms that produces goods for daily needs have a lower market risk.
Unique Risk (unsystematic risk) affects only a single firm or a small number of firms. Uncertainties about a firm's labour contracts or suppliers are part of unsystematic risk. Unique risk describes a firm's specific risk related to the market, which is assumed to be diversified away.
It has already been mentioned that you can eliminate unique risk by holding a well-diversified portfolio. Hence, *diversification* is a means of reducing risk. If we hold a large enough portfolio, unsystematic risk of individual companies cancel each other out, leaving just the systematic risk associated with each company. This will be the risk in that portfolio.
Statistics gives us the opportunity to measure risk and how to translate risk into a risk premium.

Risk-Free Interest Rate and Risk Premium

Short-term Treasury bills (e.g. 3-month U.S. Treasury bills) can be seen as risk-free investments. They bring out benchmarks of *risk-free interest rates.* As a result, the beta (describing asset riskiness) for risk-free assets is 0.
If an investor prefers to invest in an asset which is riskier than treasury bills, he is likely to demand a higher yield.
In other words, an investment in risky assets like stocks should give a higher return than that of a risk-free asset. The difference between the market risk and the risk-free rate is called *risk premium.* The risk premium can be seen as a reward for investors. This reward is what they may expect for bearing systematic risk. Therefore, it is systematic risk alone which is critical and will be higher for riskier projects than for safer projects. Risk premium represents the extra return (beyond the

risk-free interest rate) investors demand for moving their funds away from a risk-free asset to a risky asset.
The risk premium should increase with the risk aversion of an investor and the average riskiness of the investment.

Expected Return and Standard Deviation

Expected return is the weighted average of possible outcomes where the weights represent the outcome probabilities. *Standard deviation* describes the risk that expected return will or will not happen. This means that risk-free assets have a standard deviation of 0.

> Assuming an investor with the opportunity to choose between two possible investments. Say investment A and B. Both investments give you an *expected return* of 10%, but A has a *standard deviation* of 15% and therefore a wider spread of possible outcomes than B with its 7.5%. Obviously, everybody would invest in B rather than in A to avoid unnecessary risk. If you get the chance to invest in investment A or C with an expected return of 15% and a standard deviation of 10% it's clear that in that situation everybody would prefer C to A.

In real life where there are an almost unlimited range of possibilities to invest money – and not just in one asset, as in the example above. This leads us to the next section.

Portfolio Theory

Modern portfolio theory is based on an article *Harry Markowitz* released in 1952. His *Portfolio Selection Model* describes how an investor can reduce the standard deviation of portfolio returns while selecting stocks that are correlated differently.

The Capital Asset Pricing Model (CAPM)

The theory behind the CAPM and all other modern finance theories are based on the portfolio theory, developed in the early 1950`s by *Harry Markowitz*. His statement was that an investor can reduce the standard deviation of portfolio returns by choosing stocks that do not move exactly together (that are correlated differently). Furthermore, expected returns and standard deviation are the only two variables that need to be considered in an investment decision. The intuition behind the CAPM is the insurance motive of risk averse investors. The main statement of the CAPM is that one can reduce risk nicely diversifying one's portfolio.

Definitions

The assumed point of departure is that there is a competitive market and the *market portfolio* represents all assets in the economy. A broad market index like the S&P500 is used to represent the market as a whole. The *expected return* of an investment in two or more stocks is still the weighted average of the number of stocks.
A well diversified portfolio has the same *standard deviation* as the market index. A well diversified portfolio with beta 0.5 has half of the standard deviation of the market and tends to move half as much as the market.

Correlation is how strong different stocks' returns move together. Perfect positive correlation (where the correlation coefficient Corr =1) of two investments means that the returns on the investments always change at the same time in the same direction. Perfect negative correlation (Corr = -1) of two investments means that the returns on the investments always change at same time in the opposite direction. A correlation of zero (Corr = 0) means, that the returns are completely unrelated to each other.
Covariance is a measure of comovement between two random variables (how much two variables vary together). To calculate covariance, you need to have the correlation coefficient.

Beta describes the sensitivity of a particular stock's return to the return on the market portfolio r_m. Beta measures the relationship between price movements of an individual stock to the market portfolio. It is thus a measure of systematic risk. The beta of risk-free assets like treasury bills is 0. The beta of a market portfolio is 1.0. If the beta of a stock is higher than one, it is riskier than the market. One premise of the CAPM is that the risk premium moves proportionally to its beta.

For a given investment opportunity set, *efficient frontier* is a graph representing a set of portfolios that maximizes expected return at each level of portfolio risk. All other possibilities are inefficient and no one would choose such an investment. Which investment you choose that is on the efficient frontier will depend on the amount of risk you would like to bear.
The ratio of the risk premium and the standard deviation is called the *Sharpe Ratio*. This ratio measures the risk adjusted performance of investment managers. In other words, an investment manager adds value to the whole portfolio when the *Sharpe ratio* is positive. The higher the ratio is, the better the risk-return relation.

Capital Market line (Security Market line)

In the following model it is assumed that an investor can choose portfolios which consist of a share of a risk-free asset like a

treasury bill and a share of risky assets. *Capital market line* connects the risk-free asset with the risky market portfolio. This combination creates a greater set of possible portfolios and a 'new' efficient frontier line.

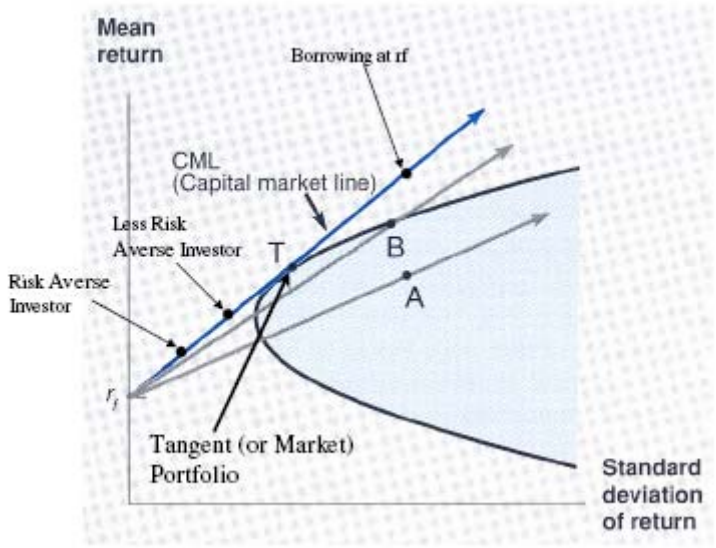

Figure: Capital Market line

An investor can borrow or lend on the risk-free interest rate. However, the most efficient portfolio would always be T (see figure above). There isn't another portfolio that makes more sense to invest in. All investors independent of their risk aversion would be on the tangent line. Where they are on the tangent line (Capital Market Line CML) will depend on the individual investor's risk tolerance. Furthermore, if it is assumed that all investors have the same information and expectations on returns, variances and correlations to the *market equilibrium* the sum of everybody's portfolio of risky assets, is the market portfolio – ie, the tangent line.

We need a measurement of systematic risk to analyze the relation between risk and expected return. This measurement should capture the relation between the market return and the return on a stock (asset). The *Beta* coefficient is an important measure of an asset's systematic risk, measured relative to the market portfolio or the comovement of a stock and the market. It's important to state that the risk premium of a stock only depends on the systematic risk, hence the higher the beta, the higher the systematic risk and the higher the expected return on the stock.
Beta can be measured as the standardized covariance between the market return and return on a stock.

Premises of the CAPM

The CAPM has, of course, some premises. One of these is that everybody holds the risk-free asset and the fully-diversified market portfolio. Only the weights (shares) determine the preference for risk. As the risk-free rate is fixed, only market risk is relevant for predicting returns. The risk-return relation (trade-off) is linear. Everybody can borrow and lend money at the same interest rate. This is unrealistic.
The composites of the expected return of an asset are:

- the pure time value of money which is measured by the risk-free interest rate,
- the reward for bearing systematic risk as measured by the market risk premium and
- the amount of systematic risk as measured by beta.

Diversification typically reduces risk. Only the non-diversifiable market risk - which is measured by beta - is important in pricing assets. CAPM combines linear risk-return trade-off with the beta to find the price of risk. The optimal risk-return trade-off is showed by the capital market line.

Critical acclaim of the CAPM

Beta isn't the only measure to predict returns as the CAPM predicts. Other measures help to predict returns (firm size, time of year, book-to-market ratios, P/E ratios) better than beta. In most cases, historical market risk premium is a proxy for the expected risk premium. The measured beta indeed is a not powerful indicator for predicting returns.
In practice, not everyone holds the full market portfolio because most diversification benefits can be obtained with relatively few assets (stocks) and mutual funds make diversification easy. Furthermore, people hold assets that are not in the stock market (real estate, human capital...) and therefore you can't reproduce the whole market. The market portfolio is unobservable so we need to use a proxy. Hence, CAPM is not testable. If you want to test it, in practice you can only test an index which represents the market. Past evidence showed that in fact, expected returns increase with beta, but not as fast as the simple version of the CAPM predicts.

Alternative Models

One alternative model for the CAPM has been developed which is adjusted for consumption. Instead of using a market beta for measuring risk, a *consumption beta* should be used. In that theory, the expected return of a stock depends on its consumption beta. There is the problem that the volatility of a stock is normally much higher than its volatility of consumption

and there aren't any sufficient methods to measure consumption. This makes this model impracticable.
Another alternative shows the *Arbitrage pricing model* ATP. The core statement is that expected return on a stock depends on more than just one market beta. For each stock there are several macroeconomic factors taken into account. The main advantage compared to the CAPM is that instead of looking at the whole market portfolio, just a sample can be looked at. The main disadvantage is that there is no single factor like market beta which defines the return on the market portfolio and that the model doesn't define the different factors.
The *Three-Factor Model* tries to abbreviate the ATP and to expand the CAPM with its market beta by two more factors that have a high degree of influence on a stock's profitability: the size of a firm and the book-to-market-ratio. Past evidence has shown that return on small stock firms and firms with a high book-to-market ratio had a higher than average profitability and therefore the three-factor model has its validity.

Each of these models defining the relationship between risk and return has its followers, but CAPM is the most widely used theory of how investors should behave when investing money in different stocks or other capital assets.

References

Principles of Corporate Finance

Richard A. Brealey, Stewart C. Myers, Franklin Allen
Ninth Edition, New York 2008, pp. 206 ff

Wikipedia

http://en.wikipedia.org/wiki/Capital_Asset_Pricing_Model
September 17th, 2008