Case Study

Marlene Jonsson, 2010-09-03

Case 1

Suppose you are working in the finance department at Akelius Fastigheter AB. Your German colleague at the business development team calls you regarding a new object that Akelius GmbH intends buying. Due to a high breakage fee on the current financing, the seller is only willing to sell if we take over the existing loans on the property.

The details of the existing loans are:

Loan 1:
Bank: Helaba
Loan amount: 3,850,000 EUR
Maturity: 2015-09-03
Interest: 5.11% p.a.
Annual interest payments
No amortisation

Loan 2:
Bank: Frankfurter Sparkasse
Loan amount: 5,000,000EUR
Interest: 1.25 % above the 3M- Euribor rate
Time to maturity: 1 year
Quarterly interest payments
No amortisation

The business developer is now asking, whether or not it is possible to take over the loans?

From your experience you know that Akelius can expect to pay a margin of 0.80 percent on a German mortgage loan and in your system you see that the 5 year market interest rate is 1.886 percent and the one year EUR interest rate is 1.189 percent.

Picture 1: 1 year and 5 year EUR market interest rate

In order to reply to your colleague you need to answer the following three questions:

- To what interest rate could Akelius finance the property
- What discount rate should be used to calculate the PV
- Calculate the present value between our funding cost and the cost of taking over the loans

Solution case 1

If Akelius should finance the property ourselves with one of our relationship banks, the interest rates would be:

	Loan 1	**Loan 2**
Margin	0.80 %	0.80 %
Market interest rate on the same duration	1.89 %	1.19 %
Akelius funding cost	**2.69 %**	**1.99 %**

Clearly, we could achieve a cheaper funding if we did not take over the loans. Therefore you recommend the business developer to reduce the price by the present value of the extra cost we would incur by taking over the loans.

The loans can be viewed as bonds because we know the size and cash flows from the mortgage loans over its duration. When calculating the present value of the difference between Akelius funding cost and the cost of taking over the loans we use the general present value formula.

$$PV= \frac{C_1}{1+r} + \frac{C_2}{(1+r_2)^2} \ldots..$$

As a discount rate we use our funding cost over the same duration as the existing loans. This can be viewed as the opportunity cost of capital.

Start	Stopp	Nominal	discount rate = our funding cost	year to maturity	interest on loan
2010-09-03	2015-09-03	3,850,000	2.69 %	5.00	5.11 %

t	interest on loan 1	our funding cost	difference	PV
1	5.11 %	2.69 %	93,324.00	90882.89
2	5.11 %	2.69 %	93,324.00	88505.62
3	5.11 %	2.69 %	93,324.00	86190.55
4	5.11 %	2.69 %	93,324.00	83936.02
5	5.11 %	2.69 %	93,324.00	81740.48
	Present value of under value			**431,255.56**

Picture 2: PV calculation of undervalue of loan 1

The present value of the under value of loan 1 is 431,255.56 EUR

We use the same method to calculate the present value of loan 2.

Start	Stopp	Nominal	disc.rate = our funding cost	Time to maturity, years
2010-09-03	2011-09-03	5,000,000	1.99 %	1.00

t	margin loan 2	our margin	under value	PV
90	1.25 %	0.80 %	5625.00	5597.37
180	1.25 %	0.80 %	5625.00	5569.88
270	1.25 %	0.80 %	5625.00	5542.52
360	1.25 %	0.80 %	5625.00	5515.30
			PV =	**22,225.08**

Picture 3: PV calculation of undervalue of loan 2

In regards to loan 2, the present value of extra cost of taking over the loan is 22,225.08 EUR.

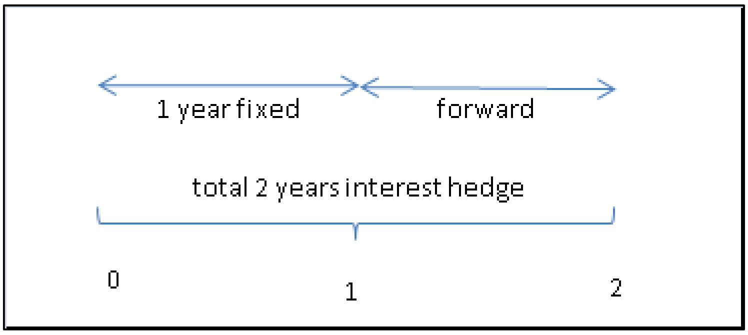
You will answer your colleague in Germany that it is possible to take over the loans but all loan agreements need to be in place with the banks before the day of possession. Secondly, you recommend that the purchase price should be reduced by the present value of the under value of the loans i.e. a reduction of 453,480.60 EUR.

Case 2:
You are the financial controller of Akelius and you are calculating the capital and interest hedges of the debt portfolio. You notice that the total volumes of short term debts are too high. As an employee in the finance department you know that the volume of interest rate hedges shorter than one year may only constitute a total volume of 25 percent of the debt portfolio. You now see that this volume has been exceeded.

You now need to come up with a solution to reduce the interest rate risk and to prolong the interest rate hedge.

One of the loans with fixed interest rate currently has a maturity of one year. You are thinking about prolonging this hedge by one year by using a forward started contract.

With a forward started contract the interest rate is fixed today for a loan to be made in the future. With a forward contract, starting after one year with a maturity of one year, you have increased the interest rate hedge to a total of two years.



The current market interest rates for the different maturities are:

EUR SWAP 1Y	1.19 %
EUR SWAP 2Y	1.24 %
EUR SWAP 3Y	1.38 %
EUR SWAP 4Y	1.55 %
EUR SWAP 5Y	1.89 %

As you need to prolong the interest hedge you ask yourself, what is the forward rate f_{2}, today?

Solution case 2:
The one-year spot rate, $r_{1,}$ and the two-year spot, $r_{2,}$ rate are known. As the forward rate is calculated from the spot rates, you can calculate the forward rate.

The general formula is:

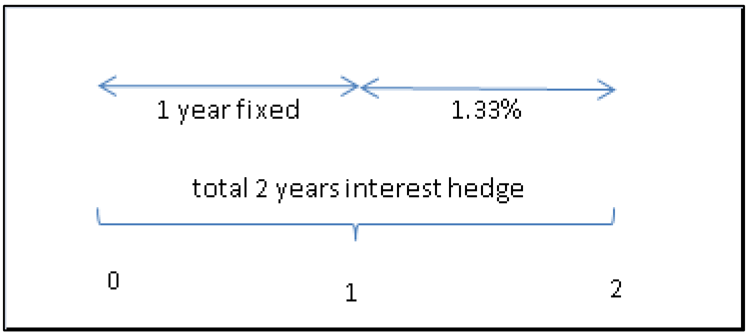
$(1+ r_2)^2 = (1+ r_1)*(1+ f_2)$

Your solution for f_2:

$$f_2 = \frac{(1+r_2)^2}{1+r_1} - 1$$

$$f_2 = \frac{(1,24)^2}{1,115} - 1 = \mathbf{1.33\ \%}$$

You discover that the forward rate, starting in one year with a maturity of one year is 1.33 percent.



You think that this is a reasonable interest rate as you expect the interest rates to increase in the future. It is a good idea to lock in a relatively low interest rate and at the same time you prolong the interest rate hedge and you will not have breached the financial policy of the company.

You propose this solution for the financial committee and they give the approval to enter into the forward agreement.

Forward rates can be calculated over several years as well and the general formula is:

$$f_n = \frac{(1+r_n)^n}{(1+r_{n-1})^{n-1}} - 1$$

f_n is the forward rate over the nth year, r_n is the n-year spot rate, and r_{n-1} is the spot rate for n-1 years.

Group work

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Valuing bonds – chapter 4

Question 1

In September 2010 you are considering buying a government bond. In your system you see a government bond that expires in September 2015. The annual coupon rate is 5 percent. The principal is EUR 1,000. Interest is paid each March and September. The market interest rate is 3 percent per year.

a) **What is the present value of the bond?**
b) **If the market interest rate unexpectedly increases, what effect would you expect this increase to have on the price of the bond?**

Question 2

Please calculate the duration and volatility on a bond where the principal is EUR 1.000 and the annual coupon rate is 4 percent and paid annually. The inflation is 2 percent and the market interest rate is 8 percent.

Question 3

On 2010-08-31 the inflation is 2 percent and the market interest rates are as following.

Year	spot rate for a *n*-year investment (% per annum)
1	2.22%
2	3.00%
3	3.30%
4	4.00%
5	4.50%

a) What are the forward rates over each of the years?
b) What is the real interest rate in year 1?

Question 4

For the next 10 years the inflation in the Euro Area is expected to be 1.50 percent. The market rates for different maturities are described below.

Year	spot rate for a *n*-year investment (% per annum)
1	2.22 %
2	3.00 %
3	3.30 %
4	4.00 %
5	4.50 %

What interest would you require on a 3 year investment in order not to lose any purchasing power?

Answers with calculations

Answer question 1a

Calculating the value of a bond is relatively easy because the size and cash flows from the bond over its life time are known.

A bond promises:

1) Regular coupon payments
2) Payment of the principal at maturity

Interest rate over any six month interval is one half of the annual interest rate. In this example, the semi-annual interest rate is 1.5 percent and the coupon payment each six months amounts to EUR 25, and there are ten of these six months periods.

The value of the bond is the present value of its cash flows i.e. the present value of the bonds coupon payments plus the present value of the repayment of the principal.

PV(bond) = (coupon payments) +PV(principal payment)

We can use the annuity formula to value the coupon payments and then we add the present value of the principal payment at maturity.

Present value of annuity = $\frac{1}{r} - \frac{1}{r(1+r)^t}$

$= 25\left[\frac{1}{0,015} - \frac{1}{0.015(1.015)^{10}}\right] + \frac{1000}{(1.015)^{10}}$ = EUR 1,092. 22

t	Coupon	PV
1	25	24.63
2	25	24.27
3	25	23.91
4	25	23.55
5	25	23.21
6	25	22.86
7	25	22.53
8	25	22.19
9	25	21.86
10	1.025	883.21
	PV =	**1092.222**

The value of the bond is EUR 1,092,22. When quoted on the screen the price of the bond is expressed as a percentage of the principal i.e. this bond trades at 109.22 percent.

The result makes sense as the bond trades at a premium. The market interest rate is less then the coupon. As newly issued bonds would give 3 percent an investor is willing to buy the bond at a premium in order to obtain the higher coupon payment.

Answer question 1b

If the market rate increases the price of the bond will decrease. Bond prices fall with a rise in the interest rate and the price will rise with the fall in interest rates.

The general principle is that a bond sells in the following ways.

- At face value if the coupon rate is equal to the market interest rate
- At a discount if the coupon rate is below the market interest rate
- At a premium if the coupon rate is above the market interest rate.

Answer question 2
The duration describes the average time to each payment
In order to calculate the duration we need to follow three steps:

1. Calculate present value of each payment

t	Coupon	PV(Coupon)	Proportion of Total Value	Proportion of Total Value x Time	
1	40	37.04	0.0506	0.0506	
2	40	34.29	0.0469	0.0937	
3	40	31.75	0.0434	0.1302	
4	40	29.40	0.0402	0.1608	
5	40	27.22	0.0372	0.1861	
6	40	25.21	0.0345	0.2067	
7	40	23.34	0.0319	0.2233	
8	40	21.61	0.0295	0.2363	
9	40	20.01	0.0274	0.2462	
10	1 040	481.72	0.6585	6.5845	
	PV =	**731.60**	100 %	**8.1184**	= Duration (years)
				7.517	= Volatility

2. Express the value of each payment in relative terms
3. Weight the maturity of each payment by its relative value

This is relatively easy done in Excel by building this table.

First we calculate the present value. Secondly, we calculate the relative value of a single payment as the ratio of the present value of the payment to the value of the bond.
As a last step we weigh the maturity of each payment by its relative value.

The bonds maturity is ten years but the weighted average time to each cash payment is only 8.12 years and this is the duration.

Bonds volatility is directly linked to its duration
Volatility (%) = $\frac{duration}{1+yield}$

In this case the volatility equals to
Volatility (%) = $\frac{8{,}1184}{1{,}08}$ = 7.517 %

This means that a 1 percentage-point variation in interest rate causes the price of the bond to change by 7.52 percent.

Answer question 3a
Because the forward rates are calculated from the spot rates, they can be determined today. We use the general formula to calculate the forward rates.

$f_n = \frac{(1+r_n)^n}{(1+r_{n-1})^{n-1}} - 1$

f_n is the forward rate over the *n*th year, r_n is the n-year spot rate, and r_{n-1} is the spot rate for *n*-1 years.

The forward rate for the first year is equal to the one-year spot rate.

$f_2 = \frac{(1.03)^2}{1.022} - 1 = 3.79\ \%$

$f_3 = \frac{(1.033)^3}{(1.03)^2} -1 = 3.90\ \%$

$f_4 = \frac{(1.04)^4}{(1.033)^3} - 1 = 6.13\ \%$

$f_5 = \frac{(1.045)^5}{(1.04)^4} - 1 = 6.52\ \%$

Year	spot rate for a *n*-year investment (% per annum)	Forward rate for *n*th year (% per annum)
1	2.22%	2.22%
2	3.00%	3.79%
3	3.30%	3.90%
4	4.00%	6.13%
5	4.50%	6.52%

Answer question 3b
The real interest rate is the interest rate expressed in terms of real goods, i.e. nominal interest rate adjusted for inflation. The formula for the real interest rate can be written as

Real interest rate = $\frac{1+\text{Nominal interest rate}}{1+\text{Inflation rate}} - 1$

Real interest rate = $\frac{1{,}022}{1{,}02}$ -1= 0.22 %

Answer question 4
As an investor you want a compensation for inflation i.e. the loss of purchasing power due to general increase in prices. If the annual prices increase by 2 percent you would like to receive 2 percent compensation in order to be able to buy the same amount of goods in the future.
By using Fisher's equation we can determine the demanded nominal interest rate.

$1+r_{nominal} = (1+r_{real})*(1+ i) -1$

(1.03*1, 02) - 1= 5.06 %